Effect of plasma single-particle excitations on the rate of nuclear reactions in the sun

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In this paper, the influence of a third body—the single particles of the plasma—on the rate of nuclear reactions in the interior of the sun is calculated using a semiclassical approach. The results suggest that increases ranging from 2% to 8% are possible for the rate of nuclear reactions in the solar core. The implications of these results for solar neutrino detectors is estimated.

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I. INTRODUCTION

Nuclear reactions in plasmas in stellar interiors are the main source of stellar energy. As a rule, the average kinetic energy of the light nuclei in the star interior is well below the height of the Coulomb barrier for the nuclear interaction. The cross section for nuclear fusion reaction is determined by a product of two factors. The first, the nuclear rection constant, has a weak energy dependence far from resonances, while the second, the penetration factor, is extremely sensitive to the ion kinetic energy and the features of Coulomb potential in the sub-barrier region.

In the solar interior, nuclear reactions proceed in the presence of a plasma of ions and electrons and one expects the plasma, because of screening and other effects, to influence the rate at which nuclear reactions occur. A number of people have studied the effect of the plasma environment on the rate of nuclear reactions ever since the pioneering work of Salpeter [1]. Calculation of nuclear reaction rates in dense plasmas have been carried out by Salpeter and Van Horn [2], Itoh, Totsuji, and Ichimaru [3], Mitler [4], Carraro, Schafer, and Koonin [5], Schramm and Koonin [6], and a number of other authors. A good review of this field is to be found in an article written by Ichimaru [7], which contains many references to original papers by other authors.

The rate of nuclear reactions between nuclear species labeled i and j is controlled by the effective potential between the nuclei at short distances. This potential, which if the nuclei were in free space would be entirely Coulomb in nature, is modified by the presence of the plasma. One of the most important contributors to this change in the potential is the screening effect due to the plasma electrons. Another important contributor is the many-body correlations that may be present in dense plasmas. The effects of the screening of the Coulomb potential by the plasma electrons have been studied in Refs. [1,2,5,8,9]. Salpeter [1], in the earliest of such studies, showed that the screening effects calculated in the adiabatic approximation where the electrons readily follow the motion of the ions increase the reaction rate between the nuclear species. Carraro *et al.* [5] showed that the screening effects are less than in the work of Salpeter [1] due to imperfect screening by the ions. The adiabatic approximation is not good for the ions, which by virtue of their large mass are not able to follow the motion of the reacting particles as readily as the electrons. In this work the dynamic effects of screening are also included in which plasma oscillations are excited in the plasma. In the work of DeWitt *et al.* [8,9] expressions for screening effects under a number of conditions are given. The size of the enhancement of the rate of nuclear reactions due to these effects is of the order of no more than a few percent. For the most important reaction in the sun, namely, $P+P \to D+e^{+}+\nu$, Ichimaru [7] has given an enhancement by 2.2% due to weak electron screening in the solar core and 4.8% due to many-body correlations among electron screened protons.

Modifications of the internuclear potential at short distances due to the nuclear reaction taking place in solids have also been studied [10]. Even the effects of including vacuum polarization on the PP reaction rate have been studied [11].

An effect that has apparently not been considered so far is the effect of a third particle, a plasma ion or an electron, influencing the nuclear reaction rate through inelastic collisions between the reacting pair and the third particle. It is the object of this paper to consider this process in a semiclassical approach.

The mechanism of the excitations we are considering is rather simple. In the initial stages prior to the nuclear reaction, the two charged particles move in the Coulomb field produced by the plasma particles. Due to this Coulomb interaction the relative motion of the two nuclear reacting particles is nonuniform. The electromagnetic field generated by these particles depends on time. This field may excite or deexcite the single-particle states of the plasma. The excitation or deexcitation of the plasma particles leads to a loss of energy or a gain of energy for the reacting particles. On the average, if during the collision with the plasma particles the probability for energy gain and loss are equal, the mean of the energy exchanged will be zero and as a result, the relative energy at which the nuclear reaction proceeds will remain unchanged. However, the mean of the square of the energy exchanged will not be zero. Such a mean square term will have the effect of giving a spread to the relative energy at which the nuclear reaction proceeds. These fluctuations in energy have a nonsymmetrical effect on the rate of the nuclear reaction between the reacting particles. When the fluctuation results in an increase in the relative energy of the reacting particles, the penetration factor will increase significantly and the reaction rate will be increased significantly. On the other hand, if the relative energy of the reacting particles is decreased due to the fluctuation, the already small penetration factor will be further reduced and hence will have a negligible effect on the rate of the nuclear reaction. The enhancement of the nuclear reaction rate due to these fluctuations arises directly from the interaction of a third body, the plasma electron or ion, with the two particles about to undergo a nuclear reaction. The effect of this additional enhancement so far has not been taken into account in arriving at the temperature of the Maxwellian distribution of the plasma ions and electrons.

In the present work we carry out a semiclassical investigation of the effect on nuclear reactions caused by the excitation of single electrons and ions in a neutral Maxwellian plasma. We use a classical description for the motion of the reacting particles in their mutual Coulomb field and calculate the probability of energy and momentum exchange with a plasma particle due to the Coulomb interaction of the plasma particle with the reacting pair. This mutual interaction of the plasma particles with the reacting particles is the source of the fields responsible for the excitation. The probability for the excitation of single electron and ion has been calculated using first-order quantum mechanical perturbation theory. It is found that the ions of the plasma, rather than the electrons of the plasma, are most effective in the energy exchange process with the reacting particles (due to the large mass of the ions) and give the most contribution to the change in the penetration factor.

II. GENERAL FORMALISM

Let us consider nuclear reaction between particles 1 and 2 embedded in a neutral Maxwellian plasma. The rate of nuclear reactions is defined by the quantity $\langle\sigma v\rangle$,

$$R = \langle\sigma v\rangle = C\, f_0 \int d\vec{V} f(V) \int dE S(E) P_0(E) \times \exp(-E/kT), \tag{1}$$

where C is the normalization constant for the Maxwell distribution, f_0 is a screening factor, $S(E)$ is a smooth function of energy, $P_0(E)$ is the penetration factor

$$P_0(E) = \exp[-2\pi\eta(E)],$$

$$\eta(E) = \frac{Z_1 Z_2 c \mu^{1/2}}{137(2E)^{1/2}},$$

μ is the reduced mass, Z_1 and Z_2 are the charge numbers of the two particles 1 and 2, E is the energy of relative motion of the two particles 1 and 2, and $f(V)$ is the Maxwellian distribution for the center of mass velocity of the two reacting particles 1 and 2.

The formula (1) is usually used in the investigations of nuclear reactions in stars. It is assumed that the energy of the particle occurring in the Maxwell distribution $C\exp(-E/kT)$ and the energy E in the penetration factor $P_0(E)$ is the same energy. In general, this may not be true, as mentioned earlier, due to the possibility of energy exchange between the reacting particles and the plasma particles in the initial stages of the nuclear reaction.

The actual penetration factor $P(E)$ that is to be used in this expression should be the penetration factor averaged over the probability distribution $\rho(E,\delta)$ for the reacting particles at relative energy E to exchange energy of an amount δ with the plasma particle. This probability distribution is explicitly calculated in Sec. III below. Let the sign of δ be positive for a gain of energy and negative for a loss of energy. This function $\rho(E,\delta)$ allows us to find out the average penetration factor through

$$P(E) = \kappa(E) \int d\delta \rho(E,\delta) \exp[-2\pi\eta(E+\delta)], \tag{2}$$

where

$$\kappa^{-1}(E) = \int d\delta \rho(E,\delta).$$

Use of formula (2) leads us to modification of (1) to

$$\langle\sigma v\rangle = C\, f_0 \int d\vec{V} f(V) \int dE\; S(E)\; P(E) \times \exp(-E/kT). \tag{3}$$

The penetration factor falls exponentially with decreasing particle energy while the Maxwell distribution extends to energies much larger than kT. Typically, one should integrate over E from kT to several times kT. Since the local deviation from thermal equilibrium is small, the range of energy transfer must be relatively small, that is, $|\delta|/kT \ll 1$ and therefore $|\delta|/E \ll 1$. Then we can represent $P(E)$ as a series expansion in δ/E using

$$e^{-2\pi\eta(E-\delta)} \simeq e^{-2\pi\eta(E)} \left[1 - 2\pi\eta(E)\frac{\delta}{2E} + [2\pi\eta(E)]^2 \frac{1}{2}\left(\frac{\delta}{2E}\right)^2\right]. \tag{4}$$

To terms of second order in δ/E, we get

$$\langle\sigma v\rangle = \langle\sigma v\rangle^0 + \langle\sigma v\rangle^1 + \langle\sigma v\rangle^2 + \cdots, \tag{5}$$

where

$$\langle\sigma v\rangle^0 = C\, f_0 \int d\vec{V} f(V) \int dE S(E) P_0(E) \times \exp(-E/kT), \tag{6}$$

$$\langle\sigma v\rangle^1 = -C\, f_0 \int d\vec{V} f(V) \int dE S(E) P_0(E) \frac{2\pi\eta(E)}{2E} \times \tilde{\delta} \exp(-E/kT), \tag{7}$$

$$\langle\sigma v\rangle^2 = C\, f_0 \int d\vec{V} f(V) \int dE S(E) P_0(E) \frac{[2\pi\eta(E)]^2}{(2E)^2} \times \frac{\tilde{\delta^2}}{2} \exp(-E/kT), \tag{8}$$

where

$$\tilde{\delta^n} = \int d\delta \delta^n \rho(E,\delta). \tag{9}$$

For reactions with E in the sub-barrier region, the factor $2\pi\eta(E) \gg 1$. Thus the small values of δ/E are somewhat compensated by factors of $2\pi\eta(E)$.

One can expect that in thermodynamic equilibrium the average exchange of energy $\tilde{\delta}$ must be small. This expectation is realized if the probabilities to gain or to lose energy δ are approximately equal. The small value of $\tilde{\delta}$ does not, however, require that $\tilde{\delta^2}$ be small. If we assume that $\tilde{\delta} = 0$, then the correction to the reaction rate will only be positive. The energy exchange between the reacting particles and the plasma particles will tend to *increase* the nuclear reaction rate.

The total correction to $R = \langle\sigma v\rangle$ can be written

$$R = \langle\sigma v\rangle = \langle\sigma v\rangle^0 (1 + \Delta R), \tag{10}$$

where the correction ΔR is

$$\Delta R = (\langle\sigma v\rangle^0)^{-1} \left(C\, f_0 \int d\vec{V} f(V) \int dE S(E) P_0(E) \times \exp(-E/kT) \frac{[2\pi\eta(E)]}{(2E)} F(E) \right), \tag{11}$$

where

$$F(E) = \left(-\tilde{\delta} + \frac{[2\pi\eta(E)]^2}{(2E)^2} \frac{\tilde{\delta^2}}{2} \right). \tag{12}$$

III. EXCITATIONS OF THE PLASMA

Let two nuclear particles with the charges $Z_1 e$ and $Z_2 e$ and the masses M_1 and M_2 collide against one another. The Coulomb interaction between the particles begins when the relative distance between the particles becomes less than a distance b of the order of the screening radius a_D (the Debye length). This distance b is then a parameter of the model. Since the only distance scale available in the plasma is the Debye length, we take the distance at which the interaction with the plasma begins to be the Debye length a_D. The magnitude of the correction we calculate depends somewhat sensitively on this parameter b and increases with an increase in b. If this parameter is chosen too large, we will no longer be able to apply the expansion in δ/E carried out in Sec. II in evaluating the penetration factor. Another relevant consideration is the mean free path for ion-ion collisions in the plasma that is of the order of several Debye lengths. The parameter b must not be larger than the mean free path for ion-ion collisions. From these considerations we take this parameter to be the Debye length $b = a_D$. In a proper calculation, using kinetic equations for the description of the plasma, this parameter would be determined self-consistently. In the absence of such a detailed treatment, it seems reasonable to assume that this distance is the Debye length a_D.

Let the relative kinetic energy of the particles 1 and 2 be E at relative distances $r > a_D$. Let us consider as initial time $t = -T$ the moment when the relative distance between the particles becomes a_D. Let the time at which the reacting particles reach the turning point $r = Z_1 Z_2 e^2/E$ be $t = 0$. We use pure Coulomb potential to describe the long distance interaction between the reacting particles.

Let us consider the energy exchange between the reacting particles 1 and 2 (coordinates $\vec{r}_1$ and $\vec{r}_2$, respectively) and the plasma environment prior to their undergoing the nuclear reaction. To get at this, we write, first of all, the potential created by particles at $\vec{r}_1$ and $\vec{r}_2$, at the point $\vec{r}_i$ in the plasma where a plasma particle i is located as

$$\phi(\vec{r}_i) = \left(\frac{Z_1 e}{|\vec{r}_i - \vec{r}_1(t)|} + \frac{Z_2 e}{|\vec{r}_i - \vec{r}_2(t)|} \right). \tag{13}$$

Let us transform to the c.m. frame of particles 1 and 2 and the relative coordinate $\vec{r}(t)$,

$$\vec{R} = \frac{M_1 \vec{r}_1 + M_2 \vec{r}_2}{M_1 + M_2}, \quad \vec{r} = \vec{r}_1 - \vec{r}_2. \tag{14}$$

In terms of these coordinates, we get

$$\phi(\vec{r}_i, \vec{r}(t)) = \left(\frac{Z_1 e}{|\vec{r}_i - \vec{R}(t) - \mu_1 \vec{r}(t)|} + \frac{Z_2 e}{|\vec{r}_i - \vec{R}(t) + \mu_2 \vec{r}(t)|} \right), \tag{15}$$

where we have introduced $\mu_1 = M_2/(M_1 + M_2)$ and $\mu_2 = M_1/(M_1 + M_2)$.

As we have mentioned before, the energy exchange may take place when the two-particle relative distance $|\vec{r}|$ is in the range $a_D > r > 0$. Therefore, inside this interval the perturbing potential energy may be taken to be, if the plasma particle has charge Z_i and mass M_i,

$$W(\vec{r}_i, \vec{r}(t)) = Z_i e[\phi(\vec{r}_i, \vec{r}) - \phi(\vec{r}_i, a_D)]. \tag{16}$$

The time dependence of W is determined by the time dependence of the relative coordinate $\vec{r}(t)$. The classical solution $\vec{r}(t)$ for the two-body problem under their mutual Coulomb interaction is well known and we take it from Landau and Lifshitz [12]. Due to the dependence of $\vec{r}$ on time, considering nuclear reactions in a neutral Maxwellian plasma, it is possible to excite two types of plasma excitations: the single electron or ion excitations and the coherent plasmon excitations. The contribution of plasmon excitation to the penetration factor has already been evaluated by Carraro *et al.* [5] and our results agree with theirs and therefore we do not consider it any further. Below we only consider the single-particle excitations in the plasma. Contributions of these to the nuclear rection rate will be estimated below.

We use (15) to estimate the probability of the excitation of the plasma single-particle states. In the first-order perturbation theory of quantum mechanics one can write

$$\rho(E,\omega_{fi}) = \left| \int_{-T}^{0} \langle f|W(\vec{r}_i,\vec{r}(t))|i\rangle e^{i\omega_{fi}t}dt \right|^2, \qquad (17)$$

where i and f label the quantum numbers of the initial and the final plasma single-particle states $\hbar\omega_{fi} = \delta = (E_f - E_i)$ and E_i and E_f are the plasma particle initial and final energies of the reacting pair (1,2). Thus $\delta = \hbar\omega_{fi}$ is the energy transfer to the plasma particle. $\rho(E,\omega_{fi})$ is the probability of the transition from the state i to the state f.

Using $\rho(E,\omega_{fi})$, one can calculate $\tilde{\delta^n}$

$$\tilde{\delta^n} = \sum_f (\hbar\omega_{fi})^n \rho(E,\omega_{fi}). \qquad (18)$$

For transitions to f in the continuum, the sum in (17) must be replaced by an integral over dE_f and $\rho(E,\omega_{fi})$ by $\frac{d\rho(E,\omega_{fi})}{dE_f}$.

The perturbing potential $W(\vec{r}_i,\vec{r}(t))$ has an effect only during a finite time interval. In this case, in order to avoid numerical problems associated with subtraction in formula (16), we have integrated (17) by parts and get the equivalent formula given by Landau and Lifshitz [13]

$$\rho(E,\omega_{fi}) = \frac{1}{\hbar^2\omega_{fi}^2} \left| \int_{-T}^{0} \frac{d\langle f|W(\vec{r}_i,\vec{r}(t))|i\rangle}{dt} e^{i\omega_{fi}t}dt \right|^2. \qquad (19)$$

To get an explicit expression for the potential $W(\vec{r}_i,\vec{r}(t))$ we use the parametric equations that describe the motion of charge Z_1 with the reduced mass μ in Coulomb field of the charge Z_2 [12]

$$x = a(\epsilon + \cosh\xi), \quad y = a(\epsilon^2 - 1)\sinh\xi$$

and

$$t = T_0(\epsilon\sinh\xi + \xi).$$

Here

$$\alpha = Z_1Z_2e^2, \quad a = \frac{\alpha}{2E}, \quad T_0 = \left(\frac{\mu a^3}{\alpha}\right)^{1/2},$$

$$\epsilon = \left(1 + \frac{2EL^2}{\mu\alpha^2}\right),$$

E is the relative kinetic energy of the reacting particles and L is the orbital angular momentum. For a reaction of nuclear fusion, the impact parameter is of the order of the nuclear radius and one can assume that the angular momentum $L = 0$ and therefore $\epsilon = 1$. We also assume that the center of mass coordinate moves with a constant velocity $\vec{V}$.

IV. SINGLE-PARTICLE EXCITATIONS IN THE PLASMA

Let us first consider the excitation of single-particle states by the potential $W(\vec{r}_i,\vec{r})$, where each of the particles has a charge Z_i and mass M_i. In the case of a Maxwellian plasma, the average kinetic energy of the reacting particles is larger than the average potential energy for the reacting particle–plasma-particle interaction. Thus we will calculate this effect in the first Born approximation and use plane waves for the plasma particle wave functions

$$|f\rangle = e^{i\vec{p}_f\vec{r}_i}$$

and

$$|i\rangle = [\rho_{M_i}(E_i)]^{(1/2)}e^{i\vec{p}_i\vec{r}_i},$$

where $\rho_{M_i}(E_i)d\vec{p}_i$ is the Maxwell distribution

$$\rho_{M_i}(E_i)d\vec{p}_i = N_i\frac{1}{(2\pi kT)^{3/2}} \times \exp(-E_i/kT)\sqrt{2}E_i^{1/2}dE_i\, d\Omega_i. \qquad (20)$$

Here N_i is the particle density for the particle type i, the volume of the system is taken to be unity, $E_i = \frac{p_i^2}{2M_i}$, and $d\Omega_i$ is the element of solid angle for $\hat{p}_i$. The integration over $\vec{r}_i$ with the plasma particle wave functions gives us

$$\langle f|W(\vec{r}_i,\vec{r})|i\rangle = \int_{-T}^{0} dt\; e^{i\omega_{fi}t}V_{fi}(t), \qquad (21)$$

where

$$V_{fi}(t) = \rho_{M_i}(E_i)v(q^2)[e^{i\vec{q}\cdot\vec{R}(t)}(Z_1e^{+i\mu_1\vec{q}\cdot\vec{r}} + Z_2e^{-i\mu_2\vec{q}\cdot\vec{r}}) - e^{i\vec{q}\cdot\vec{R}(t)}(Z_1e^{+i\mu_1\vec{q}\cdot\vec{r}_D} + Z_2e^{-i\mu_2\vec{q}\cdot\vec{r}_D})] \qquad (22)$$

and where

$$v(q^2) = \frac{4\pi Z_ie^2}{q^2},$$

$$\vec{q} = (\vec{p}_f - \vec{p}_i)/\hbar,$$

and $\vec{r}_D$ is a vector with magnitude a_D and direction of the vector $\vec{r}$. Following Landau and Lifshitz [13], the transition amplitude $a_{fi}(0)$ in first-order perturbation theory is

$$a_{fi}(0) = \int_{-T}^{0} \frac{\partial V_{fi}}{\partial t}\frac{e^{i\omega_{fi}t}}{\hbar\omega_{fi}}dt. \qquad (23)$$

After carrying out the t differentiation of V_{fi} and assuming $\vec{R}(t) = \vec{R}_0 + \vec{V}t$ we can write

$$a_{fi}(0) = \frac{ie^{i\vec{q}\cdot\vec{R}_0}(\rho_{M_i})}{\hbar\omega_{fi}}v(q^2)V(q_r,\vec{q}\cdot\vec{V},\omega_{fi}), \qquad (24)$$

where q_r is the component of $\vec{q}$ in the direction of $\hat{r}$ and

$$V(q_r, \vec{q}\cdot\vec{V}+\omega_{fi}) = \int dt e^{i(\omega_{fi}+\vec{q}\cdot\vec{V})t}\{(Z_1\mu_1 e^{-i\mu_1\vec{q}\cdot\vec{r}} - Z_2\mu_2 e^{i\mu_2\vec{q}\cdot\vec{r}})\vec{q}\cdot\frac{d\vec{r}}{dt} + (\vec{q}\cdot\vec{V})[Z_1(e^{i\mu_1\vec{q}\cdot\vec{r}} - e^{i\mu_1\vec{q}\cdot\vec{a}_D}) + Z_2(e^{-i\mu_2\vec{q}\cdot\vec{r}} - e^{-i\mu_2\vec{q}\cdot\vec{a}_D})]\}. \quad (25)$$

For transitions to f in the continuum, we have

$$d\rho(E,\omega_{fi}) = |a_{fi}(0)|^2 \frac{d\vec{p}_f}{(2\pi\hbar)^3}. \quad (26)$$

For evaluating this, we use Eq. (20) and write $d\vec{p}_f$ as

$$d\vec{p}_f = \sqrt{2} M_i^{3/2} E_f^{1/2} dE_f d\Omega_f,$$

where $d\Omega_f$ is the solid angle element in the direction of the vector $\vec{p}_f$. Using the initial direction of $\vec{r}$ as the z axis, we have

$$q_r = \vec{q}\cdot\hat{r} = p_f \cos\theta_f - p_i \cos\theta_i$$

and

$$q^2 = p_i^2 + p_f^2 - 2p_i p_f \cos\theta_{fi},$$

where

$$\cos\theta_{fi} = \cos\theta_i\cos\theta_f + \sin\theta_i \sin\theta_f \cos(\phi_i - \phi_f).$$

Here θ's represent the respective polar angles and ϕ's the respective azimuthal angles so that $d\Omega_i = \sin\theta_i d\theta_i d\phi_i$ and $d\Omega_f = \sin\theta_f d\theta_f d\phi_f$. It is possible to carry out the integrals over the azimuthal angles analytically. After these are carried out on $d\rho(E,\omega_{fi})$ we obtain

$$\frac{d\rho(E,\omega_{fi})}{dE_f} = \frac{(4\pi Z_i e^2)^2}{\hbar^2}\frac{2N_i M_i^{3/2}}{(2\pi\hbar)^3(\pi kT)^{3/2}} E_f^{1/2} \times \int_0^\infty dE_i E_i^{1/2} \times \int_0^\pi \sin\theta_i d\theta_i \int_0^\pi \sin\theta_f d\theta_f |V(q_r,\omega'_{fi}|^2 \times \frac{1}{\omega_{fi}^2}\frac{2A}{(A^2-B^2)^{3/2}}. \quad (27)$$

Here we have used $\omega'_{fi} = \omega_{fi} + \vec{q}\cdot\vec{V}$,

$$A = p_i^2 + p_f^2 - 2p_i p_f \cos\theta_i \cos\theta_f,$$

and

$$B = -2p_i p_f \sin\theta_i \sin\theta_f.$$

Using this we obtain expressions for the mean energy loss $\tilde{\delta}$ and the mean of the square of the energy loss $\tilde{\delta}^2$

$$\tilde{\delta} = \int \hbar\omega_{fi}\frac{d\rho(E,\omega_{fi})}{dE_f} dE_f \quad (28)$$

and

$$\tilde{\delta}^2 = \int (\hbar\omega_{fi})^2 \frac{d\rho(E,\omega_{fi})}{dE_f} dE_f. \quad (29)$$

Using the expression for $\frac{d\rho(E,\omega_{fi})}{dE_f}$, we finally get the expressions for the mean energy loss and the mean of the square of the energy loss as

$$\tilde{\delta} = 2N_i(Z_i e^2 a)^2 \sqrt{\frac{2}{\pi}}\left(\frac{M_i}{\hbar^2}\right)^{1/2}\frac{1}{(kT)^{3/2}}\int dE_i E_i^{1/2} e^{-E_i/kT}\int dE_f E_f^{1/2}\delta(E_f - E_i - \hbar\omega_{fi})\int \sin\theta_i d\theta_i \sin\theta_f d\theta_f \frac{1}{\hbar\omega_{fi}} \times \frac{(E_i^{1/2}\cos\theta_i - E_f^{1/2}\cos\theta_f)^2(E_i+E_f-2\sqrt{E_iE_f}\cos\theta_f\cos\theta_i)}{[(E_i+E_f-2\sqrt{E_iE_f}\cos\theta_f\cos\theta_i)^2 - 4E_iE_f\sin^2\theta_i\sin^2\theta_f]^{3/2}}|I(\xi_{\max})|^2, \quad (30)$$

$$\tilde{\delta}^2 = 2N_i(Z_i e^2 a)^2 \sqrt{\frac{2}{\pi}}\left(\frac{M_i}{\hbar^2}\right)^{1/2}\frac{1}{(kT)^{3/2}}\int dE_i E_i^{1/2} e^{-E_i/kT}\int dE_f E_f^{1/2}\delta(E_f - E_i - \hbar\omega_{fi})\int \sin\theta_i d\theta_i \sin\theta_f d\theta_f \times \frac{(E_i^{1/2}\cos\theta_i - E_f^{1/2}\cos\theta_f)^2(E_i+E_f-2\sqrt{E_iE_f}\cos\theta_f\cos\theta_i)}{[(E_i+E_f-2\sqrt{E_iE_f}\cos\theta_f\cos\theta_i)^2 - 4E_iE_f\sin^2\theta_i\sin^2\theta_f]^{3/2}}|I(\xi_{\max})|^2, \quad (31)$$

where $I(\xi_{\max})$ stands for the integral

$$I(\xi_{\max}) = \int_{-\xi_{\max}}^0 d\xi \exp i\omega'_{fi}T_0 s(\xi)\left[\frac{(\vec{q}\cdot\vec{V})T_0}{a}c(\xi)\left\{Z_1(e^{i\mu_1 q_r a c(\xi)} - e^{i\mu_1 q_r a_D}) + Z_2(e^{-i\mu_2 q_r a c(\xi)} - e^{-i\mu_2 q_r a_D})\right\} + \sinh\xi\left\{\mu_1 Z_1 e^{i\mu_1 q_r a c(\xi)} - \mu_2 Z_2 e^{-i\mu_2 q_r a c(\xi)}\right\}\right]. \quad (32)$$

Here $c(\xi) = 1 + \cosh\xi$, $s(\xi) = \xi + \sinh\xi$, $\xi_{\max}$ is defined by $a_D = a(1 + \cosh\xi_{\max})$, and $T_0 = \sqrt{\frac{\mu a^3}{\alpha}}$.

Due to the factor $\frac{1}{q^4}$ in (25), most of the contribution to the integrals (26), (29), and (30) comes from small values of q. However, there is a minimum value for q; the minimum corresponds to transferring the smallest ω_{fi}. For $|\hbar\omega_{fi}| \ll E_i$, we have

$$\frac{\hbar^2 q^2_{\min}}{M_i} = \frac{\hbar^2\omega^2_{fi} + 4\theta^2_{fi}E^2_i}{2E_i}.$$

The Debye screening limits $q_{\min}$ by the condition $q_{\min} = 1/a_D$. Thus we estimate that the important energy transfers are of the order of $[2E_i\hbar^2/(M_i a_D^2)]^{1/2}$. For conditions typical of the solar core, $E_i = 1.3k$ eV, $1/a_D = 2.5 \times 10^8$ cm^{-1}, and the energy transfer $\hbar\omega_{fi} < 300$ eV. The value of the function $|V(q_r, \omega_{fi})|^2$ reaches a maximum if the condition $\mu_1 q_r v_0 = \omega'_{fi}$ or $\mu_2 q_r v_0 = \omega'_{fi}$ is satisfied, where v_0 is the reacting particle relative velocity at a distance of separation equal to a_D. Assuming the relative energy of the reacting particles E=5 keV and E_i=1.3 keV, we get $\hbar\omega_{fi} < 200$ eV. Thus the condition $|\hbar\omega_{fi}| \ll E_i$ seems to be satisfied. In the above equations we have also used $p_f \simeq p_i[1+\hbar\omega_{fi}/(2E_i)]$, in the numerical evaluations that follow from the condition on E_f. Since these results are proportional to $p_i = \sqrt{2M_iE_i}$, it is clear that the excitations of the electrons in the plasma are not as important as the excitations of the positive ions of the plasma because of the $\sqrt{M_i}$ factor. This fact is directly related to the behavior of collision frequencies for electron-ion and ion-ion collisions in a fully ionized plasma as a function of the mass of the colliding particles.

V. DETAILS OF NUMERICAL CALCULATION

We carry out the calculation of the corrections to the rate of the nuclear reactions due to interactions with plasma ions for the processes

$$p + p, \ \ {}^3\text{He} + {}^3\text{He}, \ \ {}^3\text{He} + {}^4\text{He}, \ p + {}^7\text{Be}$$

in the core of sun. The following values of parameters are used: $kT = 1.3$ keV, $N_i = 10^{26}$ cm^{-3}, hydrogen abundance $X = 0.7$, and helium abundance $Y = 0.3$. We take the Debye radius taking account of the electrons only $a_D = \left(\frac{4\pi N_e e^2}{kT}\right)^{1/2}$. For conditions appropriate to the solar core, it has the numerical value 4×10^{-9} cm. If in defining the Debye radius the effect of the ions is also taken into account, the numerical value turns out to be about 20% smaller. As mentioned before, we take $q_{\min} = a_D^{-1}$. The minimum value of the relative distance $r_{\min}$, the turning point for classical motion of the two reacting particles, is $r_{\min} = (Z_1Z_2e^2)/E$ and therefore we take $q_{\max} = 1/r_{\min}$. The numerical value of this for $E = 8$ keV is about 2×10^{-10} cm for $Z_1 = Z_2 = 1$.

One has to perform integrations over a number of variables in order to get numerical values for the correction ΔR in Eq. (11). These variables are the variable ξ [or equivalently the time variable in Eq. (25)], the center of mass velocity V of the two reacting particles (Maxwellian distribution), the relative energy E of the two reacting particles, the initial energy E_i (Maxwellian distribution) of the ion i, the final energy E_f of the ion, and the angles θ_i and θ_f. The center of mass velocity enters the expression above only through the combination $\vec{q}\cdot\vec{V}$ in Eq. (32). The frequency ω_{fi} appears in the combination $\omega'_{fi} = \omega_{fi} + \vec{q}\cdot\vec{V}$, which amounts to a Doppler shift of the frequency due the motion of the center of mass. This effect is clearly a small one because the magnitude of the velocity of center of mass motion is so much smaller than the magnitude of the relative velocity. Hence we have adopted an approximate way in which to take this effect into account. We assume that the center of mass velocity has the most dominant component in the direction of the initial relative position vector $\hat{r}$; then $\vec{q}\cdot\vec{V} = q_rV$ and we integrate over the restricted Maxwellian distribution in the one-dimensional variable V. Then the integrations over the other variables were carried out numerically following standard methods.

In an effort to get some feeling for the magnitude of the correction Δ, one might consider evaluating $\tilde{\delta}$ and $\tilde{\delta}^2$ at one particular value of $E_i = kT$ of the Maxwellian distribution. For this particular value of E_i one finds that $\tilde{\delta} = \tilde{\delta}^2/(2kT)$ holds numerically. The expression for ΔR in (11) contains in its integrand the exponential factor $\exp[-E/kT - 2\pi\eta(E)]$ in addition to $F(E)$, depending on a combination of $\tilde{\delta}$ and $\tilde{\delta}^2$. At the maximum for the exponential factor that occurs at $E = E_1$, where $E_1^{3/2} = 2\pi Z_1Z_2c\sqrt{\mu}kT/(2 \times 137\sqrt{2})$, the factor in parentheses vanishes for $E = E_1$. Thus, in the integration over E (if the Maxwellian was replaced by a single energy $E_i = kT$), the integrand is exactly zero at $E = E_1$. It has opposite signs for contributions to the integral for $E < E_1$ and for $E > E_1$. Because of this, cancellations occur in the evaluations of the integral in (11). For other values of E_i of the Maxwellian involved, similar cancellations occur and therefore one has to be very careful in the evaluation of these integrals numerically. Bearing this fact in mind, we have carried out these numerical integrations paying special attention to the cancellations.

In the first step of numerical calculation the quantities $|I(\xi_{\max})|^2$ as a function of q_r, ω_{fi}, and V are stored. In calculating $\tilde{\delta}$ and $\tilde{\delta^2}$, numerical integration over 80 values of E_f, over 50 values of θ_i, and over 50 values of θ_f is carried out. Also 20 values of V are chosen. For each value of θ_i and θ_f the momentum transfer q_r is calculated and the corresponding value of $|I(\xi_{\max})|^2$ is determined with an interpolation for the needed values of q_r. The integration over the collision relative energy E and averaging over the Maxwellian distributions for E_i are carried out over 20 values of energy each.

It turns out that the value of the integral (32) is rather sensitive to the choice of the distance at which energy exchange with the plasma begins. Generally speaking, this must be obtained from the self-consistent kinetic equations in the plasma. However, using a_D for this distance, we get a lower estimate for the value of the integral.

The results for $\tilde{\delta}$ and $\tilde{\delta}^2$ and ΔR are given in Tables I and II, respectively, for each of the pairs of nuclear

TABLE I. Dependence of various quantities as a function of the relative energy E of the reacting particles for plasma particles i=H,He for a fixed $E_i = kT$. Column 2 gives the quantity $P = \exp[2\pi\eta(E) - (E/kT)]$, columns 3–5 and 6–8 give, respectively, $\tilde{\delta}$, $\tilde{\delta}^2$, and $G(e)$ for the H components and the He components of the plasma. Here $G(E)$ stands for the integrand of the E integral involved in the expression for ΔR, Eq. (11).

		Plasma H component			Plasma He component		
E (keV)	$P(\times 10^8)$	$\tilde{\delta}$ (keV)	$\tilde{\delta}^2$ (keV^2)	$G(E)$	$\tilde{\delta}$ (keV)	$\tilde{\delta}^2$ (keV^2)	$G(E)$
2	2.7	0.05	0.134	0.835	0.087	0.227	1.44
3	23	0.059	0.152	0.23	0.098	0.256	0.39
4	61	0.062	0.160	0.07	0.103	0.268	0.12
5	93	0.062	0.158	0.019	0.105	0.27	0.032
6	103	0.061	0.156	-0.0004	0.105	0.273	-0.0004
7	95	0.06	0.155	-0.008	0.105	0.273	-0.0132
8	76	0.06	0.155	-0.01	0.105	0.27	-0.018
9	55	0.06	0.155	-0.014	0.104	0.27	-0.02
10	37	0.06	0.155	-0.012	0.104	0.27	-0.015
11	24	0.06	0.155	-0.011	0.104	0.27	-0.014

reacting particles mentioned above, influenced by hydrogen and helium ions of the plasma. To see whether the results obtained by us for the numerical values of $\tilde{\delta}$ are reasonable, we present an estimate for it following general arguments on neutral Maxwellian plasma. In a neutral plasma, the rate at which energy exchange between ions occurs in the plasma is controlled by the mean free path for ion-ion collisions [14]

$$l \sim \frac{(kT)^2}{4\pi e^4 N_i L},$$

where L is the Coulomb logarithm $L = \ln(\frac{a_D kT}{e^2})$. Using values for N_i,kT, mentioned earlier appropriate for the solar core, we get

$$l \sim \frac{10^{-7}}{ZZ'} \text{ cm},$$

where Ze is the charge of one of the reacting particles and $Z'e$ is the charge of the plasma ion. The parameter $\tau = l/v$ characterizes the relaxation time for the reacting particle energy to relax to the equilibrium value. Using this as a guide, we can estimate that during the passage of the reacting particles through a relative distance a_D, the energy exchange δ should be of the order $\delta = \frac{a_D}{l}E$, where E is the initial relative energy of the reacting particles. Taking this energy to be about 5–6 keV, the amount of energy exchange is roughly 100–300 eV. The numbers in Table I for $\tilde{\delta}$ are in good agreement with expectations based on general arguments about the scale associated with these energy exchanges and gives us confidence in our numerical work.

VI. IMPLICATION OF THE RESULTS FOR SOLAR NEUTRINO DETECTORS

The corrections to the rates of nuclear reaction in the sun core are represented in Table II. One can see that the effects of the energy exchange play a more important role for the $p+p$ reaction. The most probable energy of this reaction is about of 6 MeV, while for the other reactions considered this energy is 3–4 times larger. Therefore, the relative contribution of the energy exchange with the plasma particles for the $p+p$ reaction (the typical scale of energy transfer is of the order of a few hundred eV) is larger in comparison with the other reactions.

The data of Table II do not answer one important question, namely, what are the expected corrections to the observed rates for solar neutrino detectors? Unfortunately, the computer codes for the standard solar model are not available to us, so we cannot carry out detailed self-consistent calculations. However, using the results of Turck-Chièze and Lopes [15] and of Bahcall and Ulrich [16], we can give two estimates for the effect of the increased nuclear reaction rate on solar neutrino detectors.

First, let us use the work of Turck-Chièze and Lopes [15] for this estimate. These authors have considered the

TABLE II. Corrections to the rate of nuclear reactions in the solar core.

Reaction type	Contribution of the H component (%)	Contribution of the He component (%)	Total ΔR (%)
$p+p$	3.3	4.7	8
$^3He + {}^3He$	2.3	4	6.3
$^3He + {}^4He$	1	3	4
$p + {}^7Be$	0.8	1.5	2.3

effects of variations in the rates of nuclear reactions in the solar core on the theoretical predictions for the different solar neutrino detectors. For example, the relations between the correction ΔS_{11} to the rate of $p+p$ reaction and the theoretical predictions for the observed neutrino detector rates may be written

$$\Delta R_W = -1.6\Delta S_{11},$$
$$\Delta R_{\rm Cl} = -1.2\Delta S_{11},$$
$$\Delta R_{\rm Ga} = 0.54\Delta S_{11},$$

where ΔR_W, $\Delta R_{\rm Cl}$ and $\Delta R_{\rm Ga}$ are the corrections to water, chlorine, and gallium detectors, respectively.

The work of Turck-Chièze and Lopes [15] allows us to estimate the total correction to the expected rates from all the considered reactions as

$$\Delta R_W = -10\%, \quad \Delta R_{\rm Cl} = -8\%, \quad \Delta R_{\rm Ga} = 4\%.$$

Second, let us use the work of Bachall and Ulrich [16]. They have given analytical formulas giving the fluxes of pp, ^{7}Be, and ^{8}B neutrinos as

$$\Phi(pp) \approx S_{11}^{0.14} S_{33}^{0.03} S_{34}^{-0.06} L_\odot^{0.73} (Z/X)^{-0.08} (\mathcal{A})^{-0.07},$$

$$\Phi(^7{\rm Be}) \approx S_{11}^{-0.97} S_{33}^{-0.43} S_{34}^{0.86} L_\odot^{3.4} R_\odot^{0.22} (Z/X)^{0.58} (\mathcal{A})^{1/3},$$

$$\Phi(^8{\rm B}) \approx S_{11}^{-2.6} S_{33}^{-0.4} S_{34}^{0.81} S_{17}^{1.0} L_\odot^{6.8} R_\odot^{0.48} (Z/X)^{1/3} (\mathcal{A})^{1/3},$$

where S_{11}, S_{33}, S_{34}, and S_{17} are the rates for $p+p$, ^{3}He+^{3}He, ^{3}He+^{4}He, and p+^{7}Be reactions, $L_\odot$ is the solar luminosity, and $R_\odot$ is the solar radius.

Assuming that $L_\odot$, $R_\odot$, (Z/X), and the age $\mathcal{A}$ do not depend on the rate of nuclear reactions, we get

$$\Delta\Phi(pp) = 1\%, \quad \Delta\Phi(^7{\rm Be}) = -7\%, \quad \Delta\Phi(^8{\rm B}) = -19\%.$$

Using these data we can estimate the corrections to the theoretical predictions for the solar neutrino detectors

$$\Delta R_W = -19\%, \quad \Delta R_{\rm Cl} = -16\%, \quad \Delta R_{\rm Ga} = -3\%.$$

Indeed, we are showing these results as only qualitative estimates. First of all, the analytical expressions of Bahcall and Ulrich [16] may be applicable only for small variations of input values of the nuclear reaction rates, of the order of 1%, and it may not be appropriate to estimate the effect as we have done. Another point to be kept in mind is that we should emphasize again that our description of the effect is a semiclassical one and in this a crucial role is played by the choice of the distance at which energy exchange with the plasma starts to occur. Because of these considerations we do not attempt a quantitative comparison between theory and experiment. The purpose of this work is only to draw attention to the presence of the energy exchange mechanism with the plasma having an effect on the rates of nuclear reactions and providing some estimate of the size of the effect. The tendency of this effect of increase in the rate of nuclear reactions seems to be particularly large for the $p+p$ reaction. As a result, the flux of ^{7}Be and ^{8}B neutrinos will decrease and the magnitude of this change can be as much as 10%.

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